Analysis of the vibrational dynamics of a two-axle vehicle with real road surface roughness amplitude

Phân tích dao động cơ học của xe hai trục với biên độ mấp mô mặt đường thực tế

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Abstract: The calculation of automobile oscillations is crucial for evaluating stability and vertical dynamic safety during vehicle operation. In this study, a two-axle vehicle dynamics model is developed using differential equations to compute the vehicle's dynamic oscillations. The Matlab-Simulink tool is utilized to solve these differential equations and determine the dynamic response of the two-axle vehicle system subjected to external excitations. These excitations are measured and simulated based on real road surface irregularities recorded by a Walking Profiler device. The research results have successfully developed a vehicle dynamics model that considers the random roughness of the road surface during vehicle operation. The random roughness factor of the road surface is incorporated into the simulation model as a primary excitation factor affecting the characteristic outcomes of vehicle oscillations.

Keywords: *Roughness amplitude; Simulation model; Two-axle vehicle; Vibrational dynamic; Walking profiler.*

Tóm tắt: Tính toán dao động ô tô có ý nghĩa quan trọng, và cơ sở để đánh giá độ ổn định, an toàn động lực học theo phương thẳng đứng trong quá trình hoạt động của ô tô. Trong nghiên cứu này mô hình động lực học ô tô 2 trục được xây dựng với hệ phương trình vi phân trong tính toán động lực học dao động của ô tô trên cơ sở khai thác công cụ Matlab – Simulink để giải hệ phương trình vi phân để tìm đáp ứng động lực học cơ hệ dao động xe 2 trục với ngoại lực kích động được đo đạc và đưa vào mô phỏng từ mấp mô mặt đường thực tế được đo bằng máy đo Walking Profiler. Kết quả nghiên cứu đã phát triển được mô hình tính toán động lực học ô tô chuyển động có xét đến độ mấp mô ngẫu nhiên của mặt đường khi ô tô hoạt động. Và đã đưa yếu tố mấp mô ngẫu nhiên của mặt đường vào mô hình mô phỏng như một yếu tố kích động ảnh hưởng chính đến các kết quả đặc trưng của dao động ô tô.

Từ khóa: *Dao động cơ học; Độ mấp mô bề mặt; Mô hình mô phỏng; Thiết bị đo mấp mô; Xe hai trục.*

1. Introduction

The study of automobile vibrations examines the oscillations caused by road surface irregularities and explores noise reduction solutions during vehicle operation. It is essential for evaluating vertical dynamic stability, safety, and ride comfort for both passengers and cargo. The findings from these studies greatly aid in researching the durability of bridges and roads during real-world usage when vehicles travel over these infrastructures. Two critical components of automobile vibration research are the criteria for evaluating vibrations and the development of vehicle vibration models.

Due to the need for enhancing and optimizing automotive suspension systems, the factors causing vehicle vibrations are now considered more comprehensively. The impact of vibrations is evaluated not only in terms of ride comfort for passengers and cargo but also regarding their detrimental effects on road surfaces. Consequently, the demand for research into vehicle dynamics and control dynamics has led to the development of more diverse, complex, and accurate vibration models. An overview of vibration research encompasses the following four key areas:

-Vibration evaluation criteria;

-Vibration models, including physical models (equivalent vibration diagrams) and mathematical models (differential equation systems);

-Excitation functions;

-Vibration experiments.

2. Two-axle vehicle dynamic model

The vehicle oscillation model represents a complex multi-body system, integrating multiple masses interconnected by elastic elements. The study becomes more facile and manageable when dividing the system into subsystems, thereby modularizing during both modeling and programming phases. This approach of structural decomposition is also conducive to simulation-based methodologies within Matlab Simulink.

On a vehicle, the suspended masses and unsuspended masses are interconnected through the suspension system, which possesses elastic and damping characteristics. In a model, the suspension system can be treated as a submodule. From an oscillation perspective, the wheels also function as elastic elements, thereby constituting subsystems.

Starting from the real-world object (the actual vehicle), it is essential to establish a physical model (an equivalent oscillation diagram) from which mathematical equations can describe the oscillation state of the components (the vehicle), referred to as a mathematical model. In the context of vehicle oscillations, specifically disregarding small amplitude and high-frequency oscillations, the vehicle can be viewed as a multi-body system. A multi-body system in mechanics consists of a finite number of bodies interconnected by forces under dynamic constraints [1].

Thus, a vehicle oscillation system is modeled as a physical representation (equivalent oscillation diagram) comprising masses and inertial moments interconnected via joint forces. The oscillatory behavior is described using a system of second-order differential equations.

The model of a two-axle vehicle (such as a sedan, van, or small to medium truck) traveling at constant speed on a road can be described as a two-axle, four-degree-of-freedom vehicle model (planar model). The vehicle body is assumed to be a rigid beam. This beam has a mass mmm, equal to half the total mass of the vehicle body, and a moment of inertia Iy, equal to half the vehicle body's moment of inertia about its horizontal axis. The front and rear wheels have masses m1 and m2 respectively. Tire stiffness is represented by different parameters kt1 and kt2, reflecting the typical higher stiffness of rear tires compared to front tires. In a simplified model, kt1 could be assumed to equal kt2. Tire damping is significantly lower compared to shock absorber damping. [1,2].

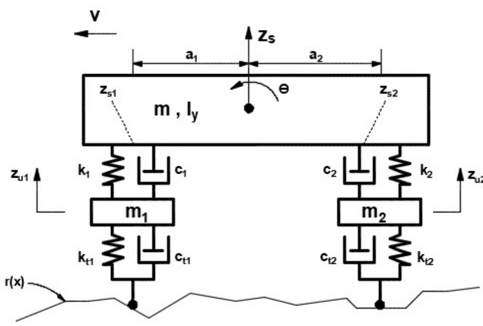

Figure 1. Two-Axle Vehicle Oscillation Model

To find the equations of motion for a vehicle oscillation model, we use the Lagrangian method. The equations defining the kinetic and potential energies of the system are as follows: [3]

$$K = \frac{1}{2}m\dot{z_s}^2 + \frac{1}{2}m_1\dot{z}_{u1}^2 + \frac{1}{2}m_2\dot{z}_{u2}^2 + \frac{1}{2}I_z\dot{\theta}^2 \quad (1)$$

$$V = \frac{1}{2}k_{t1}(z_{u1} - z'_{u1})^2 + \frac{1}{2}k_{t2}(z_{u2} - z'_{u2})^2 + \frac{1}{2}k_{t1}(z_s - z_{u1} - a_1\theta)^2 + \frac{1}{2}k_{t2}(z_s - z_{u2} - a_2\theta)^2 \quad (2)$$

And the dissipation function is:

$$D = \frac{1}{2}c_1\left(\dot{z}_s - \dot{z}_{u1} - a_1\dot{\theta}\right)^2 + \frac{1}{2}c_2\left(\dot{z}_s - \dot{z}_{u2} - a_2\dot{\theta}\right)^2 \quad (3)$$

And applying the Lagrangian method:

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{q_r}}\right) - \left(\frac{\partial K}{\partial q_r}\right) + \frac{\partial D}{\partial \dot{q_r}} + \left(\frac{\partial V}{\partial q_r}\right) = f_r ;$$

With $r = 1,2 \dots 4$ (4)

The equation of motion for the oscillation model:

$$m\ddot{z}_s + c_1\left(\dot{z}_s - \dot{z}_{u1} - a_1\dot{\theta}\right) + c_2\left(\dot{z}_s - \dot{z}_{u2} - a_2\dot{\theta}\right) + k_1(z_s - z_{u1} - a_1\theta) + k_2(z_s - z_{u2} - a_2\theta) = 0 \quad (5)$$

$$I_z\ddot{\theta} - a_1c_1\left(\dot{z}_s - \dot{z}_{u1} - a_1\dot{\theta}\right) + a_2c_2\left(\dot{z}_s - \dot{z}_{u2} - a_2\dot{\theta}\right) - a_1k_1(z_s - z_{u1} - a_1\theta) + a_2k_2(z_s - z_{u2} - a_2\theta) = 0 \quad (6)$$

$$m_1\ddot{z_{u1}} - c_1\left(\dot{z}_s - \dot{z}_{u1} - a_1\dot{\theta}\right) + k_{t1}(z_{u1} - z'_{u1}) - k_1(z_s - z_{u1} - a_1\theta) = 0 \quad (7)$$

$$m_2\ddot{z_{u2}} - c_2(z_s - z_{u2} - a_2\theta) + k_{t2}(z_{u2} - z'_{u2}) - k_2(z_s - z_{u2} - a_2\theta) = 0 \quad (8)$$

These equations can be rearranged into matrix form:

$$[m]\ddot{z_s} + [c]\dot{z_s} + [k]z_s = F \quad (9)$$

With:

$$z_s = \begin{bmatrix} z_s \\ \theta \\ z_{u1} \\ z_{u2} \end{bmatrix};$$

$$[m] = \begin{bmatrix} m & 0 & 0 & 0 \\ 0 & I_z & 0 & 0 \\ 0 & 0 & m_1 & 0 \\ 0 & 0 & 0 & m_2 \end{bmatrix};$$

$$[c] = \begin{bmatrix} c_1 + c_2 & a_2c_2 - a_1c_1 & -c_1 & -c_2 \\ a_2c_2 - a_1c_1 & c_1a_1^2 + c_2a_2^2 & a_1c_1 & a_2c_2 \\ -c_1 & a_1c_1 & c_1 & 0 \\ -c_2 & -a_2c_2 & 0 & c_2 \end{bmatrix};$$

$$[k] = \begin{bmatrix} k_1 + k_2 & a_2k_2 - a_1k_1 & -k_1 & -k_2 \\ a_2k_2 - a_1k_1 & k_1a_1^2 + k_2a_2^2 & a_1k_1 & a_2k_2 \\ -k_1 & a_1k_1 & k_1 + k_{t1} & 0 \\ -k_2 & -a_2k_2 & 0 & k_2 + k_{t2} \end{bmatrix};$$

$$F = \begin{bmatrix} 0 \\ 0 \\ z'_{u1}k_{t1} \\ z'_{u2}k_{t2} \end{bmatrix}$$

The system of differential equations above constitutes the mathematical model describing the oscillations of a two-axle vehicle. To solve these equations, numerical simulation methods can be utilized, such as using Matlab – Simulink software. After solving the equations and simulating the oscillations of the two-axle vehicle, we can accurately determine the amplitude, velocity, and acceleration of the vehicle body's center of gravity.

Considering the actual road surface as a continuous beam with an average profile surface r(x), the Simulink diagram simulating the planar oscillations of the two-axle vehicle is constructed as depicted in Figure 2. The planar vertical oscillation model of the two-axle vehicle comprises:

- Wheel element,
- Unsprung mass element with one degree of freedom,
- Suspension system element,
- Sprung mass element with two degrees of freedom and two excitation forces.

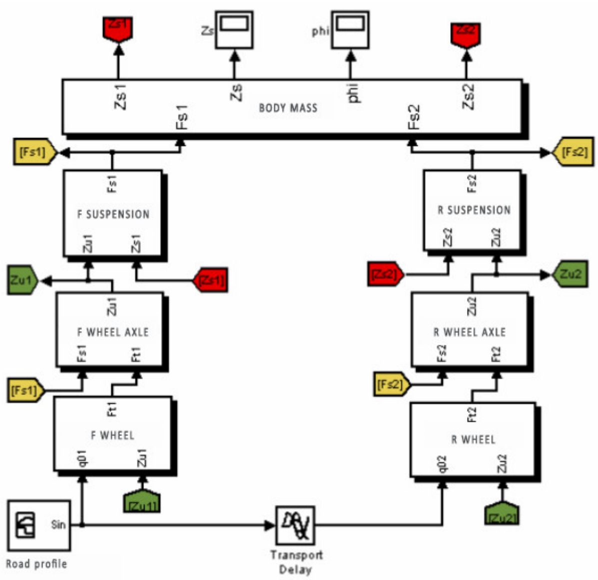

Figure 2. Simulink Diagram for Simulating Vehicle Oscillations with a Two-Axle Model

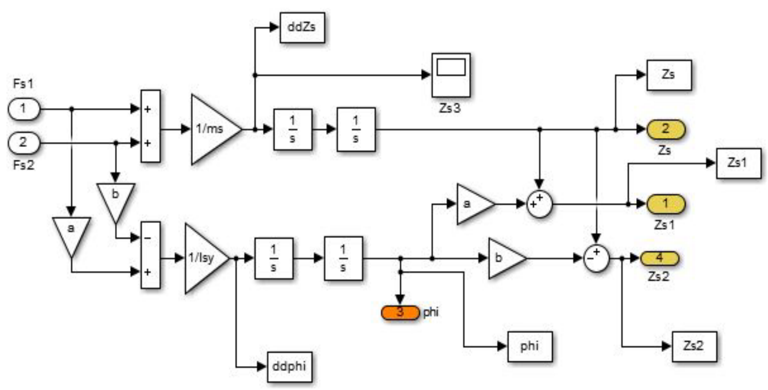

Figure 3. Vehicle Body Module

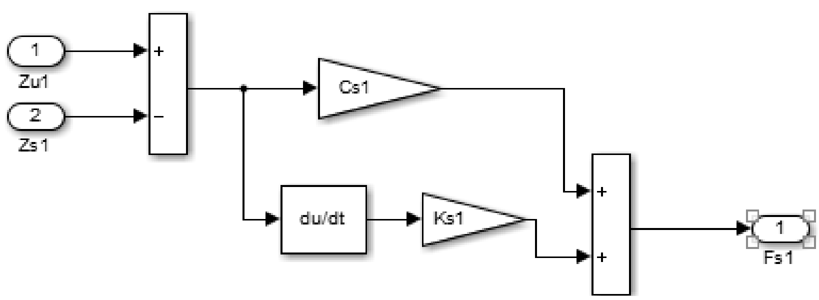

Figure 4. Front Suspension System Module

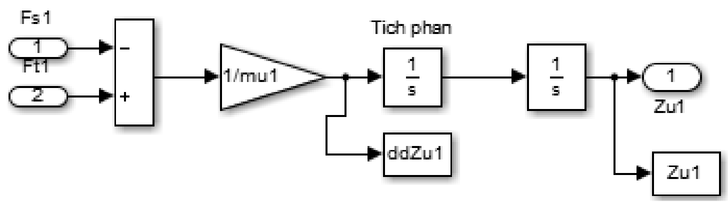

Figure 5. Front Axle Module

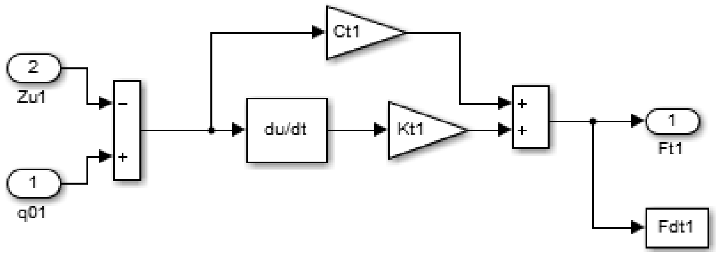

Figure 6. Front Wheel Module

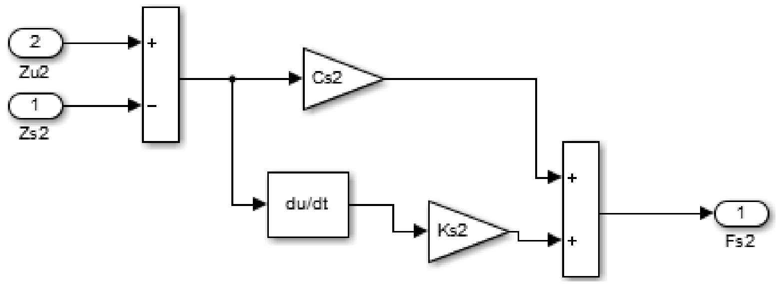

Figure 7. Rear Suspension System Module

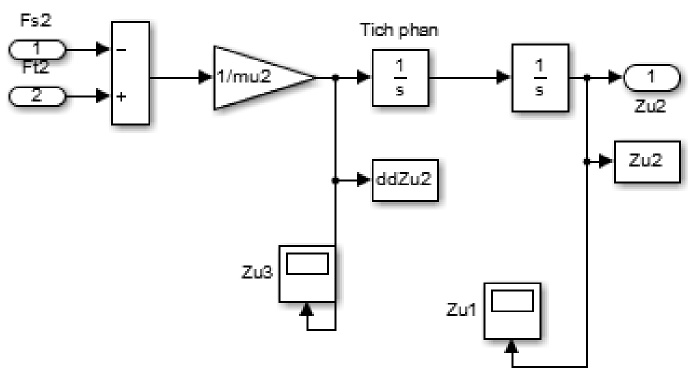

Figure 8. Rear Axle Module

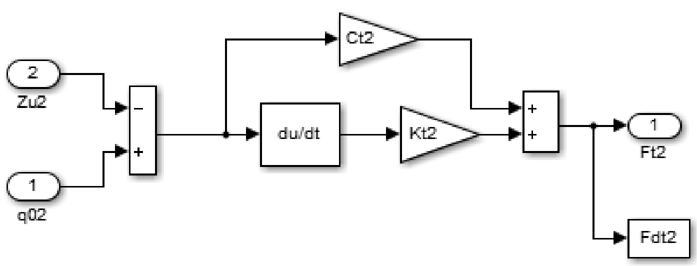

Figure 9. Rear Wheel Module

Table 1. Input parameters to calculate vibration of 2-axle vehicle.

Parameter/Meaning	Symbol	Value
Half of body mass	m [kg]	2290
Mass of a front wheel	m_1 [kg]	185
Mass of a rear wheel	m_2 [kg]	215
Stiffness of the front suspension	k_1 [N/m]	50000
Stiffness of the rear suspension	k_2 [N/m]	38000
Damping coefficient of the front suspension	c_1 [Ns/m]	10000
Damping coefficient of the rear suspension	c_2 [Ns/m]	11000
Stiffness of the front wheel	k_{t1} [N/m]	460000
Stiffness of the rear wheel	k_{t2} [N/m]	500000
Damping coefficient of the front wheel	c_{t1} [Ns/m]	460
Damping coefficient of the rear wheel	c_{t2} [Ns/m]	500
Half of body lateral mass moment of inertia	I_y [kg.m^2]	2771
Distance of C from front axle	a_1 [m]	1.38,
Distance of C from rear axle	a_2 [m]	1.26
Body vertical motion coordinate	z_s [m]	
Front wheel vertical motion coordinate	z_{u1} [m]	
Rear wheel vertical motion coordinate	z_{u2} [m]	
Body pitch motion coordinate	Θ [rad]	
Road excitation at the wheel	$r(x)$ [m]	

3. Experimental measurement of the real road surface profile

The real road surface profile was measured using a Walking Profiler machine (refer to Figure 10.). Measurements were performed three times for accuracy, and the final result is the average of these three measurements (Figure 11.). The largest peak-to-valley height measured (between the highest peak and the lowest point on the surface) was 45 mm. These measurements of the road surface profile serve as the basis for simulating a random rough surface profile.

Figure 10. Walking Profiler for measuring road surface profiles

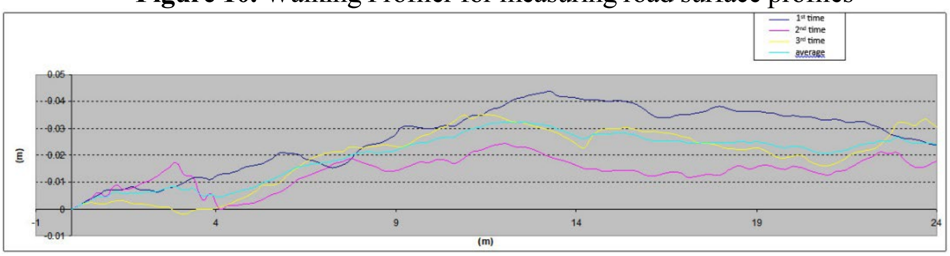

Figure 11. Graph illustrating the road surface texture profile

After detrending and conducting statistical analysis, the statistical characteristics of the surface profile are as follows:

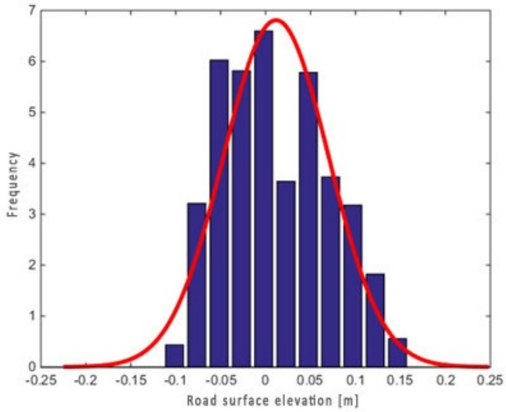

Figure 12. Probability density function of the road surface elevation

Group division for statistical data processing: number of groups n. [4]

n=1+3,32*log10(N)

where N is the total number of random data points (peak-to-valley heights).

Statistical expectations: Mean (mean value): Mean=0,0116

- Standard deviation: Std=0,0586
- Skewness: Skn=0,2983

The road surface is designed as Type A with class 1 according to ISO 8608 standards. Applying the research on describing the road surface texture as a random function, we obtain the power spectral density and the form of the road surface texture function as shown in Figure 13 [5, 6].

The vehicle oscillation model is based on the Newton-Euler decomposition principle to construct the system of differential equations of motion. A Simulink-Matlab simulation is developed for a two-axle vehicle using specific data. The road surface roughness is treated as the input excitation load. By solving the problem using Matlab Simulink with the specified model and data, as shown in Figure 14, Figure 15, Figure 16.

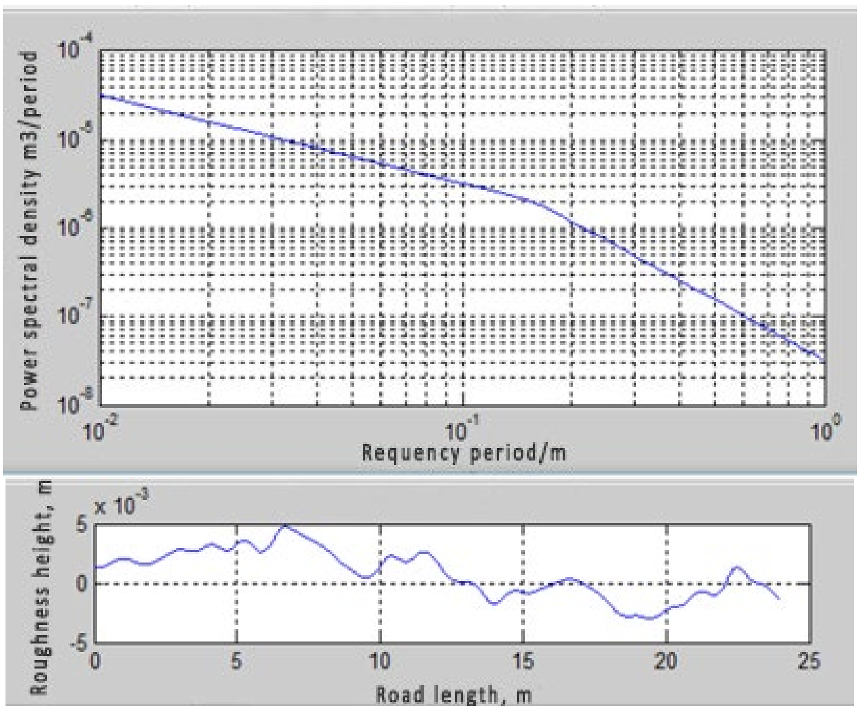

Figure 13. Power Spectral Density and Level 1 Road Surface Roughness.

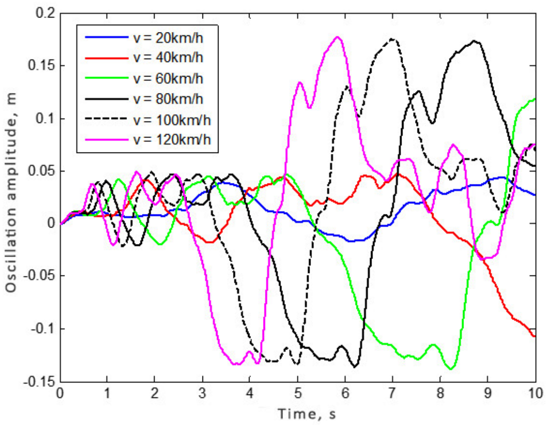

Figure 14. Comparison of oscillation amplitudes Zs at the vehicle's center of gravity at different speeds.

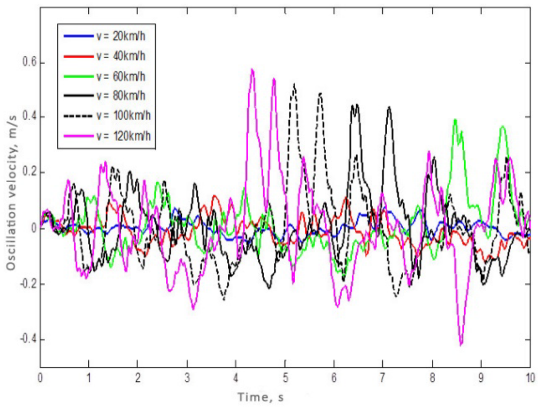

Figure 15. Comparison of oscillation velocities at the vehicle's center of gravity at various speeds.

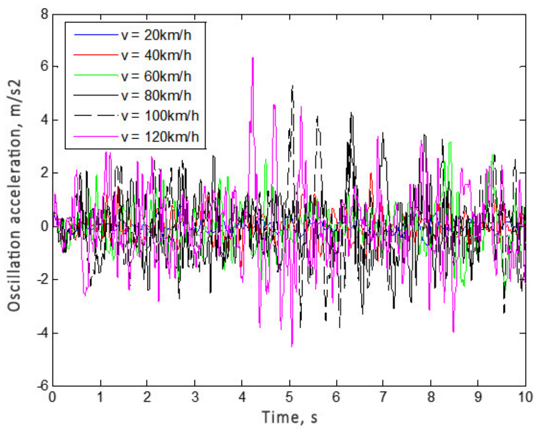

Figure 16. Comparison of oscillation accelerations at the vehicle's center of gravity at various speeds.

The vehicle oscillation model is based on the Newton-Euler decomposition principle to establish the system of differential equations of motion. A Simulink-Matlab simulation is developed for a two-axle vehicle using specific data, with road surface roughness considered as the input excitation load. The results show:

At speeds of 20 km/h and 40 km/h, the oscillation amplitudes and peak vertical accelerations remain within safe limits, indicating that the vehicle's suspension system functions effectively and ensures stability at lower speeds.

At speeds of 60 km/h and 80 km/h, the peak vertical accelerations reach the warning threshold, suggesting that at intermediate speeds, the suspension system approaches a critical point where careful monitoring and control are needed to avoid negative impacts on vehicle stability and passenger comfort.

At speeds of 100 km/h and 120 km/h, the peak vertical accelerations exceed the intervention threshold, indicating a higher risk of instability or loss of control at higher speeds. This highlights the need for improvements in suspension system design or the implementation of intervention measures to maintain safety and stability at high speeds [7].

4. Conclusion

To date, research on the dynamic response of moving vehicles through numerical simulations has garnered significant attention. Vehicle dynamic models are increasingly being refined to enable relatively accurate simulations of the system's behavior.

This thesis advances the development of vehicle dynamics models by considering the road surface roughness as an excitation factor that alters the oscillatory characteristics of the system.

Key findings and contributions of the thesis include:

1. Development of a Vehicle Dynamics Model: The thesis has developed a dynamic model for moving vehicles that incorporates the random roughness of the road surface.

2. Generalized Differential Equations: It has established a set of generalized differential equations for vehicle dynamics applicable to different vehicle models.

3. Inclusion of Random Road Roughness: The thesis has integrated random road surface roughness into the simulation model as an excitation factor affecting the oscillatory characteristics. The statistical properties of the random roughness were derived from survey measurements of actual road conditions.

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